

Math 10a
Practice Midterm 1 #1

1. The volume of an expanding cloud of gas starts at 10 L and multiplies by some factor m each second.
 - (a) In terms of m and number of elapsed seconds t , write down an expression for the volume of the gas after t seconds.
 - (b) If, after 100 seconds, the cloud of gas has doubled in size, what is m ? No need to simplify your answer.
 - (c) How long until the gas reaches 50 L?
 - (d) Let $V(t)$ denote the volume as a function of time. What is the instantaneous rate of change of $V(t)$ at $t = 0$?

2. Differentiate:

- (a) $f(x) = \frac{1}{1+x^2}$.
- (b) $f(x) = \sqrt{\sqrt{x}}$.
- (c) $f(x) = \cos^2\left(\frac{x^2}{2}\right)$.
- (d) $\ln(1 + \sqrt{x})$.

3. A hyperbola has an equation given by

$$\frac{x^2}{4} - \frac{y^2}{9} = 1.$$

What is the equation to the tangent line to the hyperbola at $(2\sqrt{2}, 3)$?

4. $\sqrt[3]{2}$ is close to $\frac{5}{4}$. Use one iteration of Newton's method to improve on this guess.
5. (a) What is the 3rd degree Taylor polynomial of $\sin(x)$ centered at $x = 0$?
(b) Use your answer to part (a) to estimate $\sin\left(\frac{1}{4}\right)$.